Let us take as our point of departure, the following passage from Lyndon LaRouche’s main article, a passage that has specific bearing upon Mozart’s composition of the Fantasy in C minor, K. 475, in May 1785:

“A further refinement is required. The mind hears the inversion of any interval (e.g., C-E-G heard as G-E-C), to such effect that a simple Lydian scale is derived as an inversion of a C-minor, F♯ pivotted scale. The effort to bring the intervals represented by the scale indicated by the inversion, [into coherence] with the scale which has been inverted, introduces a further degree of refinement of the well-tempering. Add, the inversion heard across the polyphonic parts to the inversions generated within each part, and a further refinement is introduced.”

Mozart opens the K. 475 Fantasy with a bare statement of just such a “C-minor, F♯ pivotted scale” (Figure 5.1). But before we plunge into the work itself, let us first see precisely what kinds of inversions are required to derive a “simple Lydian scale” from it. Let us represent the leading features of the original scale as C-E♭-(F♯)-G. The intervals described are an ascending minor third, followed by an ascending augmented second, and then an ascending half-step. Now, using C as our pivot, invert the direction of the intervals from ascending, to descending. The result is C-A-(G♭/F♯)-F, a kind of F major with a G♭ hovering just above the tonic. Finally, taking F as our point of departure, take the C-G♭/F♯ interval and reverse the direction again, projecting it upward (Figure 5.2). The result: F-A-(♭B)-C, a simple Lydian scale. That the mind hears such relations implicitly, is proven beyond a doubt by the power of Mozart’s K. 475 Fantasy.

Another unique property of Lydian intervals should also be touched upon before we begin to grapple with Mozart’s compositions in detail. Disregarding different spellings for the moment (in actual composition, they are crucial for the shaping of tone), one quickly discovers that there exist six, and only six, unique Lydian intervals in the well-tempered domain, namely: C-F♯, D♭-G, D-A♭, E♭-A♯, E♭-B♭, and F♭-B♭.
These intervals are not “all alike,” but flow in a certain hierarchy as we move away from C-F♯, upon which the entire well-tempered domain is pivoted. After C-F♯, come the pair F-B♭ (as we saw derived above), and close after it, E♭-A♭, by virtue of the close relation of the E♭ major scale to the C major/minor mode. (Beethoven, in the opening of Act II of his opera Fidelio, goes so far as to tune the two kettle-drums to precisely those two tones, E♭ and A♭!!) Next we have another pair, each pivoted on other members of the C-major triad: E♭-B♭, and G-D♭. And finally, there is D♭-A♭, the most “distant” Lydian.

Just as with the hierarchy of the register-shifts of the six species of bel canto-trained voices, so the mind hears this implicit hierarchy of Lydians. But the relationship between Lydians and register-shifts is far more profound than that: If one makes a list of the six Lydian intervals, and cross-grids this with the six human voice species, one finds that every Lydian interval crosses at least one vocal register of each of the six voice species. Or, put another way: The introduction of any one Lydian interval implies a register shift for every species of singing-voice. Other intervals larger than the Lydian also share this property; but the next smaller interval, the fourth, does not have that property. E.g., for the ascending fourth C-F, the soprano never shifts registers across this interval.

The Lydian interval, therefore, represents the minimum action required to move into the domain of multiply-connected, polyphonic vocal registration. And conversely, the interval of a fourth represents the threshold of that domain, just as the soprano’s and tenor’s F♯ is the threshold of the F♯ register shift.

It was implicit principles such as these—and not the mere form of fugal writing—that struck Mozart like a bolt of lightning when he was introduced to J.S. Bach’s works by Gottfried van Swieten and his circles, beginning around 1782. From this standpoint, let us take yet another look at one of the works which Mozart studied intensively, the six-part Ricercar from Bach’s A Musical Offering. Focus on the end of the opening statement (measures 9-11 in Figure 5.4): As the second voice enters, the first voice continues with a sequence of ascending fourths. On the first beat of measure 11, the new voice creates a Lydian interval C-F♯ with the first voice; this is followed by a series of descending fourths. As Bach proceeds through each successive variation, he uses inversion to increase the density of sequences of fourths; in measure 89 (Figure 5.5), he also introduces a rhythmic shift, such that the first note of the pair is shifted from the “strong” beats of the measure (beats 1 and 2), to the off-beats (beats 1½ and 2½). The density of rising and descending fourths reaches its greatest in measures 180ff., where the derived cross-voices all focus to form the Lydian interval C-F♯.

The revolution in Mozart’s mind, consisted in making these discovered principles into the explicit, primary subject of the composition, using the new method of motivic thorough-composition as pioneered by his friend Joseph Haydn. The fruit of Mozart’s compositions of those years, quickly ripened into a new modal method of composition, in which the Lydian-register shift plays the pivotal role. In his explorations, Mozart stuck close to the key of C minor in order to maximize his focus on working out the implications of his discovery. Examples of these efforts are the Fugue in C minor for Two Pianos, K. 426 (which he reworked five years later for string quartet), and his unfinished Great Mass in C minor, K. 427.

But greatest of them all, is his C minor Fantasy K. 475, and its companion work, the Sonata in C minor for solo piano, K. 457. Without additional commentary, Mozart had both works published together; to the educated audiences of his day, he did not need to explicitly state the obvious point, that the Fantasy was his own investigation into the principles employed in his composition of the earlier sonata. And, as we shall see below, the Fantasy indeed picks up exactly where the Sonata leaves off. (Beethoven, in many of his later works, took this approach one step further, interrupting the composition in mid-stream if necessary, in order to force the audience to turn their attention away from the rich externalities of the composition, to the discoveries of principles implied therein. One of the most accessible examples of this, is the baritone’s recitative “O Freunde, nicht diese Töne!” (“Not these tones, my friends!”) in the fourth choral movement of his Ninth Symphony.)

In order to even reach the doorknob of Mozart’s Fantasy, therefore, we must first, at very minimum, take a “guided tour” of the companion sonata. For, just as it is impossible to grasp the significance of Gauss’s discovery of the orbit of Ceres without working through the problem step-by-step, so it is with Mozart’s discoveries here. The toil will be well worth it.

[text continues on page 69]
The K. 457 Sonata

The K. 457 sonata consists of three moments: the first a “Molto allegro” in C minor; the second, an extended “Adagio” in E♭ major, and the third, an “Allegro assai”—to which Mozart later added the word “agitato”—once again in C minor.

The first movement opens with a simple ascending C minor arpeggio, played forte, followed by a contrasting piano sequence consisting of a descending fifth G-C (inversion of a fourth), and a descending diminished seventh A♭-B♭—the same interval which marks the opening motivic statement of Bach’s A Musical Offering (Figure 5.6). The first Lydian interval is formed with that B♭, but its significance does not go much beyond its cadential function. This is followed by a restatement of the same sequence, but beginning on G, with the B♭ changed to a B♮, in order to keep it in the mode of C minor, once again ending in a cadentially-used F-B♭ Lydian. The paradoxes only really begin with the following, second poetic couplet of the opening (measures 9-12): against an ostinato G-g in the bass, are descending fourths in two voices. The first voice descends in half-steps: G-F♯-F♮-E♭-D—again an explicit reference to the descending line in the opening of Bach’s A Musical Offering. And, as with Bach’s work, it is introduced as a mezzosoprano voice. The second descending fourth is E♭-D-C-B♮—a diminished fourth, introduced in the soprano voice but with strong registral implications for the tenor voice. (This is probably why Beethoven focussed on this very interval in his Sonata Op. 5, No. 2 for Piano and Violoncello, since the predominant vocal reference for the ’cello is the tenor voice.) Mozart repeats this double figure an octave higher, in such a way that the mezzosoprano line is now sung by a soprano voice—once again in keeping with Bach’s treatment. To conclude the opening idea, Mozart repeats the descending half-step interval a♭-g♭ twice, as an inversion of the implicitly stated G-A♭ of the opening measures 3 and 4.

Let us now skip to the second part of
the first movement, beginning at the double-bar on measure 75 (Figure 5.7). Here the opening arpeggio is re-introduced, but now in C major. Mozart almost immediately moves up to the implied inversion of C major, which is F minor, with many mentions of the Lydian Bb. The passage ends on measure 98 with the same  as in the first section, only shifted down two octaves to precisely the location where the bass voice shifts between the second and first registers; the interval is suspended against the Bb (Figure 5.8).

The original C-minor arpeggio is now restated, but now it is enriched with a canon focusing on Ab (measures 118-120), and a sudden shift into Db, with the implicit, rising figure C-D  being heard as the inversion of the falling C-Bb (Figure 5.9). The density of Lydians and inversions reaches a high-point at measure 125, where C-F is superimposed right on top of Bb-F. (Beethoven clearly recognized the significance of this passage, and in Mozart’s honor, made them into the high-point of the Kyrie section of his Mass in C major, Op. 86.)

The extended “Adagio” of the sonata’s second movement creates the necessary attention span for working through the paradoxes introduced in the first movement. As is the hallmark of Mozart’s motivic thorough-composition, no additional musical material is really added. The opening (Figure 5.10) is a descending fourth, b3'-f', ending with a rising fourth e5”-c5”, and then a descending combination of both, ending with a very prominent dwelling on the Lydian e5’-a7’. The introduction is then repeated, but with an added element: a descending diminished fourth e5”-b7’, which was also briefly referenced in the first movement but not explored. In this movement, it is explored exhaustively, through multiple inversions.

The implications of the first movement’s A7-G are now also intensively worked through in a section in Ab (Figure 5.11). Against an ostinato A7-Ab (as opposed to G-g in the first movement), two other voices are at work: the first descends from e5-d5-c-Bb, while in the
second, Mozart simply reverses the order of the first and second pair of notes, thus: c’-b⁰-b⁰⁰-d⁰. And—should we be surprised by now?—Beethoven celebrates his great teacher here as well, by quoting this passage “verbatim” in the second “Adagio cantabile” movement of his C minor sonata for Piano, Op. 13 (Pathétique).

Mozart now introduces the most “distant” Lydian, A⁰-D⁰, into the musical fabric (measure 27), and in short order (measure 32, Figure 5.12), leads us into what can only be described as “ontological surprise”: the same A⁰ material is begun again, but in G⁰. Aha! F!! [text continues on page 72]
A series of arpeggios (measures 38-40) leads us to g”, a♭”, a♯”, and b♭”, and back “home” to the opening statement, but now highly ornamented (Figure 5.13). The registral shifts implied by E♭-A♭ and A♭-D♭ are now brought out clearly, both in measure 49, with the sudden drop into the “chest” register (Figure 5.14), and in the two grand scales in measures 51 and 52, moving from the soprano’s high b♭” to the bass’s low A♭, and then back up a high a♭”-a♭”, with each note suspended by a fermata (Figure 5.15). [text continues on page 73]
The third and final movement of the sonata is organized as a kind of rondo, where the same theme repeatedly returns, unaltered. Each time, one moves farther afield, only to be jolted back, as in Poe’s refrain “Nevermore.” In the opening (Figure 5.16), we are back to the same material as the first two movements, but now the F♯ is more prominently ending the phrase on F♯-g♯.

A very rapid descending fourth f♯-c♯ is followed by repeated g’s, ending with a series of double Lydian, arpeggiated chords focusing on B♭; followed by a pregnant silence. And quietly (measure 26), we hear what everything seems to have been driving at all along: e♭-f♯-g♯-a♭—the beginnings of Mozart’s explicitly F♯-pivotted C minor. However, an uneasy paradox remains: by placing the e♭ above the f♯, implicitly in the soprano voice, there is no register shift. This absence of register-shift is what makes the passage so haunting each time it recurs in the rondo. Indeed, the paradox is never really satisfactorily resolved in the sonata itself—which is probably why Mozart added “agitato” to the description, and which is decidedly part of his motivation for composing the Fantasy later on.

We move into a passage in E♭ major, with its A♭ Lydian reference (Figure 5.17), harkening back to the second movement, ending with a sequence of rising fourths B♭-C-D-E♭ in three different voices, all harkening back to the opening of the second movement. [text continues on page 74]
The rondo returns, and now we move to the F minor of the first movement, with the added figure c"-d"-c" bringing to mind the first movement’s g-a♭ and a♭-g (Figure 5.18). The suspicion is confirmed when the same material is repeated in C minor (measures 168-169), and g'-'a♭'-'g' is heard explicitly. Yet another shift, and this is inverted as F♯-G in five different voices (Figure 5.19).

A new round of the rondo, and further aspects of the preceding movement are brought into focus. Then, with a final reference to F minor, Mozart proceeds to his remarkable coda, which starkly presents two sequences. The first is a descending scale (measures 293-300) which seems oddly bent out of shape: c"-b♭"-a♭"-f"-e"-d"-c"-(b♭') (Figure 5.20). But in the domain of the mind’s hearing of implicit inversions, it is not “bent” at all, for, if one inverts it, it is a simple C major sequence: C-D-E-G-A-B-C. But there is a note missing: the fourth degree, F. The final sequence (Figure 5.21) solves that problem, not with F, but with F♯. First c"-♭♭" is stated high in the soprano’s fourth register, followed by a huge leap to an F♯’ at the very bottom of the piano’s range, below the regular singing ranges; and ended with G'-A♭', then F♯’, and a final C. Which is precisely where Mozart begins in the Fantasy. [text continues on page 75]
The K. 475 Fantasy

It is impossible to reduce the opening of the Fantasy to a “theme.” The opening represents a multiple dimensionality, whose implications only begin to unfold as one ponders over the kaleidoscopic sequence of all sections of the Fantasy, taken as a whole; the which, obviously, can only be done if one has worked through it in a good deal of detail.

The most immediately evident feature is Mozart’s inclusion of the F♯ into the very first measure (Figure 5.22). This F♯ has a register shift “written all over it”: The shift from E♭ to F♯ is a register shift for the four most common of the six voice species: soprano, mezzo-soprano, tenor, and baritone.

The Promethean challenge posed by this included F♯ is then underlined, quietly, in the second measure, with a sequence of double Lydians, all revolving around F♯-G. It is essential that these two pairs be performed as true appoggiaturas, with the emphasis on the first member of each pair. If this is done correctly, the second pair leads as a cross-voice into the silence of the measure’s final beat; if it is performed incorrectly, the pairs degenerate into perfunctory, meaningless cadences.

But we must also consider the following dimensionality: The opening measure has three parts: (1) the opening C; (2) the four subsequent notes E♭-F♯-G- A♭, which describe a rising fourth which has been oddly “bent out of shape”; and (3) the leap back into the “chest register” on C-B♭, with its implications for the simple F-based Lydian.

And, as yet another dimensionality, we should consider the downward, G-pivotted inversion of the opening sequence: G-E♭-D♭-C- G- A♭, with its included prominent G-D♭ Lydian. Already in measure 4, this interval is brought out explicitly as well, and beginning with measure 6, the opening, descending C-B♭ is inverted into a rising C-D♭. In measure 10, Mozart re-inverts this relationship into B♭ again, this time referencing the full B♭ mode. But we remain there only momentarily, as the bass descends by half-steps from there down to G♭ (measure 15). And, surprise!
In measure 16, G♯ is transformed into F♯, in a passage which is an unmistakable reference to the surprising G♯ passage in the second movement of the sonata. Only here in the Fantasy, it took Mozart only sixteen measures to bring us there!

F♯ remains the focus of the following measures, leading to measure 25 (Figure 5.23), where repeated F♯s move into the key of D major—a key which was entirely absent from the K. 457 sonata. It is also striking that the Lydian G♯ is entirely absent from opening bars of this D major section; Mozart’s focus here, rather, is on the motivic thorough-composition associated with C-B♯, but this time transformed into descending whole-steps, such that the entire melody is constructed from nothing but these descending and ascending whole steps.

The situation is entirely reversed, with a vengeance, in the following “allegro” passage (measures 36ff.), where a repeated E-F♯ is played against repeated d”-g♯” (Figure 5.24). The same sequence is then repeated one step lower, on D-E♭ and e”-f♯”, in such a way that we are led directly into an F major/minor section with a stress on the B♭-F Lydian (Figure 5.25). But not for long, as we are pressed back to a grand sequence of double-Lydian arpeggios, against a descending bass line from F♯ to the lowest F♯ on the piano keyboard (Figure 5.26). We go a half-step lower still, to F♯, and then swing with arpeggios and scales from the bottom to the top of the vocal range, finally descending slowly to two long, held notes, e♯” and e♭”—a reference to the held a♭” and a♯” in the sonata’s second movement.

[text continues on page 77]
Quite an adventure! We have already worked through the content of the entire sonata. But it is far from over. Mozart now focuses on an ascending sequence of four descending fourths, such that the sequence itself describes a fourth (measures 86-88, Figure 5.27), after which he repeats the sequence down an octave, and then down another octave. Of all the subsections of the Fantasy, Mozart tarries here the longest; the descending fourths are filled out with half-steps, are complemented by parallel sixths, and are repeated in all registers. All of this occurs in B♭ major. [text continues on page 78]
Finally, Mozart shifts the same material to C minor (Figure 5.28). What a difference! Instead of the fourth, we get a diminished fourth e–d–c–b which we have encountered a number of times before. And now Mozart launches into a “Più allegro” section that consists of nothing but sequences and inversions, with successive rising fourths soon supplanted by rising Lydians, of which every type is referenced. To make sure the discovery is not lost, Mozart cycles through a series of summary cadences, each punctuated by double-Lydian arpeggios, which recapitulate every mode referenced in the work: C major/minor, F minor, D major, G minor, finally ending, in measure 154 (Figure 5.29), on C, F, and D played simultaneously, followed by g against f. Two “false” cadences against F, which we can finally grasp the actual course of Mozart’s discoveries.

The foregoing “guided tour” of the C minor Fantasy is only the first, but necessary step in grasping the discovered principles as a unity in the mind’s eye. Through multiple comparisons of the successive transformations across sections—and not necessarily in their temporal sequence!—we can finally grasp the actual course of Mozart’s discoveries.